4.1 Coordinates and Scatter Plots

Goals
- Plot points in a coordinate plane.
- Draw a scatter plot and make predictions about real-life situations.

VOCABULARY

Coordinate plane

Ordered pair

x-coordinate

y-coordinate

Graph

Scatter plot

Example 1 *Plotting Points in a Coordinate Plane*

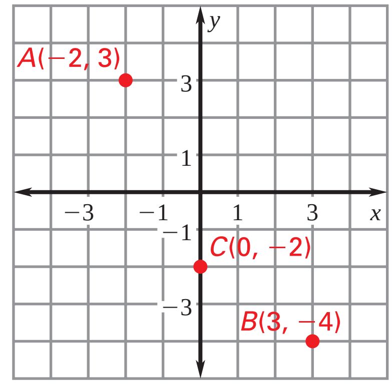
Plot the points $A(-2, 3)$, $B(3, -4)$, and $C(0, -2)$ in a coordinate plane.

Solution

To plot the point $A(-2, 3)$, start at the ________. Move ___ units to the _____ and ___ units ____ .

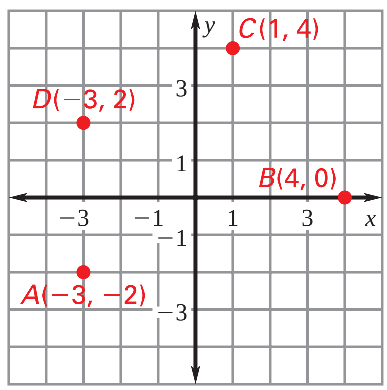
To plot the point $B(3, -4)$, start at the ________. Move ___ units to the ______ and ___ units _______ .

To plot the point $C(0, -2)$, start at the ________. Move ___ units to the ______ and ___ units _______ .



Checkpoint **Plot the points on the same coordinate plane.**

1. $A(-3, -2)$ **2.** $B(4, 0)$ **3.** $C(1, 4)$ **4.** $D(-3, 2)$



Example 2 *Making a Scatter Plot*

NCAA Basketball Teams The number of NCAA men's college basketball teams is shown in the table.

Year	1995	1996	1997	1998	1999	2000
Teams	868	866	865	895	926	932

a. Draw a scatter plot of the data.

b. Describe the pattern of the number of men's basketball teams.

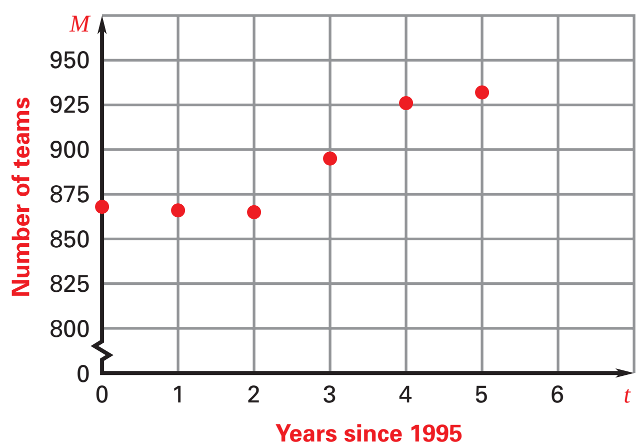
Solution

a. Let *M* represent ______________________________ . Let *t* represent ______________________________.

Because you want to see how the number of teams changed over time, put *t* on the __________ axis and *M* on the ________ axis.

Choose a scale. Use a break in the scale for the number of teams to focus on values between _____ and _____ .

NCAA Men's Basketball Teams



b. From the scatter plot, you can see that the number of men's basketball teams in the NCAA was ________________ for three years and then began to ____________________ .

4.2 Graphing Linear Equations

Goals
- Graph a linear equation using a table or a list of values.
- Graph horizontal and vertical lines.

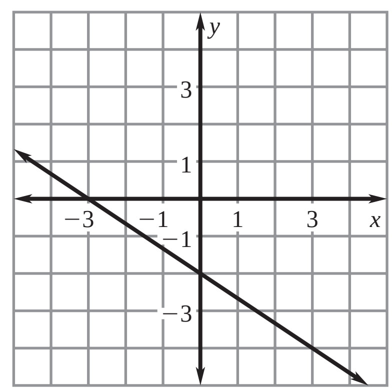
VOCABULARY

Solution of an equation

Graph of an equation

Example 1 ***Verifying Solutions of an Equation***

Use the graph to decide whether the point lies on the graph of $2x + 3y = -6$. Justify your answer algebraically.



a. (3, −4) **b.** (−4, 1)

Solution

a. The point (3, −4) ___ on the graph of $2x + 3y = -6$. Therefore, (3, −4) ___ a solution of $2x + 3y = -6$. You can check this algebraically.

$2x + 3y = -6$ **Write original equation.**

$2(\_\_) + 3(\_\_\_) \stackrel{?}{=} -6$ **Substitute ___ for *x* and ____ for *y*.**

$\_\_\_ \ \_\_ \ -6$ **Simplify. ______ statement.**

b. The point (−4, 1) _______ on the graph of $2x + 3y = -6$. Therefore, (−4, 1) _______ a solution. You can check this algebraically.

$2x + 3y = -6$ **Write original equation.**

$2(\_\_\_\_) + 3(\_\_) = -6$ **Substitute ____ for *x* and __ for *y*.**

$\_\_\_\_ \ \_\_ \ -6$ **Simplify. ____________ statement.**

GRAPHING A LINEAR EQUATION

Step 1 Rewrite the equation in ______________, if necessary.

Step 2 Choose a few values of ___ and make a _______________.

Step 3 Plot the points from the table of values. A line through these points is the _______ of the equation.

Example 2 ***Graphing an Equation***

Use a table of values to graph the equation $x + 4y = 4$.

After an equation is rewritten in function form, it is easier to make a table of values.

1. Rewrite the equation in function form by solving for y.

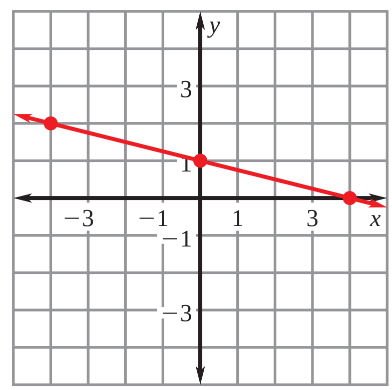
$x + 4y = 4$ **Write original equation.**

$4y =$ ________ **Subtract ___ from each side.**

$y =$ ________ **Divide each side by ___.**

2. Choose a few values of x and make a table of values.

Choose x.	-4	0	4
Evaluate y.			



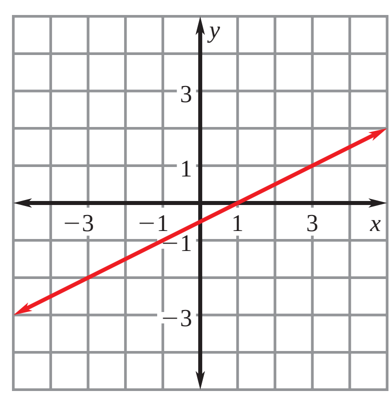
The solutions are

$(-4,$ ___$)$, $(0,$ ___$)$, and $(4,$ ___$)$.

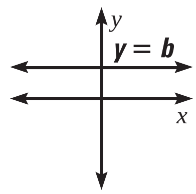
3. Plot the points and draw a line through them.

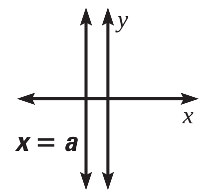
Checkpoint **Complete the following exercise.**

1. Use a table of values to graph the equation $x - 2y = 1$.



EQUATIONS OF HORIZONTAL AND VERTICAL LINES





In the coordinate plane, the graph of $y = b$ is a __________ line.

In the coordinate plane, the graph of $x = a$ is a ________ line.

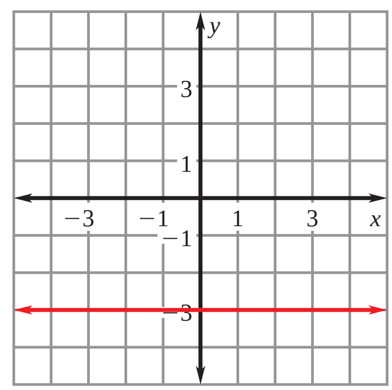
Example 3 *Graphing $y = b$*

Graph the equation $y = -3$.

Solution

The equation does not have x as a variable. The y-value is always -3, regardless of the value of x. For instance, here are some points that are solutions of the equation:

$(-3, -3), (0, -3), (3, -3)$.



The graph of the equation is a __________ line ___ units _______ the x-axis.

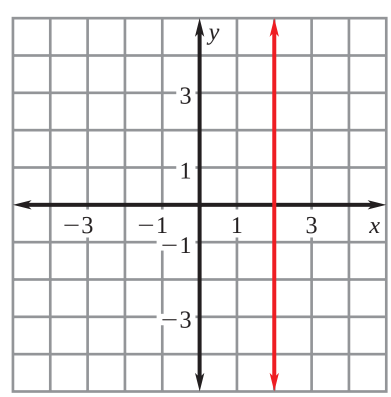
Example 4 *Graphing $x = a$*

Graph the equation $x = 2$.

Solution

The equation does not have y as a variable. The x-value is always 2, regardless of the value of y. For instance, here are some points that are solutions of the equation:

$(2, -3), (2, 0), (2, 3)$.



The graph of the equation is a ________ line ___ units to the ______ of the y-axis.

4.3 Quick Graphs Using Intercepts

Goals
- Find the intercepts of the graph of a linear equation.
- Use intercepts to make a quick graph of a linear equation.

VOCABULARY

x-intercept

y-intercept

Example 1 ***Finding Intercepts***

Find the *x*-intercept and the *y*-intercept of the graph of the equation $-3x + 4y = 12$.

1. To find the *x*-intercept of $-3x + 4y = 12$, let $y =$ ___ .

$-3x + 4y = 12$	**Write original equation.**
$-3x + 4($ ___ $) = 12$	**Substitute ___ for *y*.**
_____ $= 12$	**Simplify.**
$x =$ _____	**Solve for *x*.**

The *x*-intercept is _____ . The line crosses the *x*-axis at the point (_____ , ___).

2. To find the *y*-intercept of $-3x + 4y = 12$, let $x =$ ___ .

$-3x + 4y = 12$	**Write original equation.**
$-3($ ___ $) + 4y = 12$	**Substitute ___ for *x*.**
____ $= 12$	**Simplify.**
$y =$ ___	**Solve for *y*.**

The *y*-intercept is ___ . The line crosses the *y*-axis at the point (___ , ___).

Example 2 *Making a Quick Graph*

Graph the equation $3x + 2.5y = 7.5$.

Solution

Find the *x*-intercept.

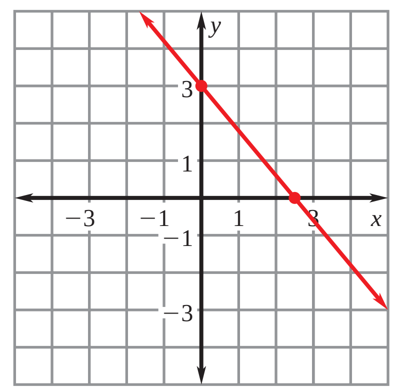
$3x + 2.5y = 7.5$	**Write original equation.**
$3x + 2.5(\_\_\_) = 7.5$	**Substitute ___ for *y*.**
$\_\_\_\_ = 7.5$	**Simplify.**
$x = \_\_\_\_$	**Solve for *x*. The *x*-intercept is ____ .**

Find the *y*-intercept.

$3x + 2.5y = 7.5$	**Write original equation.**
$3(\_\_\_) + 2.5y = 7.5$	**Substitute ___ for *x*.**
$\_\_\_\_\_ = 7.5$	**Simplify.**
$y = \_\_\_$	**Solve for *y*. The *y*-intercept is ___ .**

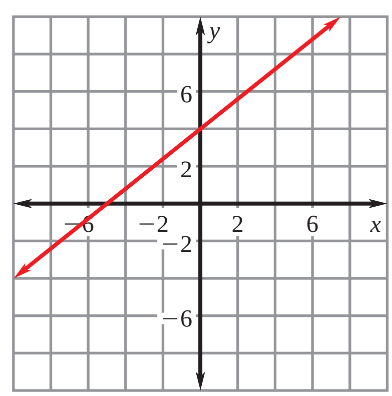
Only two points are needed to determine a line.

Plot the points (____ , 0) and (0, ___) and draw a _____ through them.



Checkpoint Complete the following exercise.

1. Find the *x*-intercept and the *y*-intercept of the graph of the equation $-4x + 5y = 20$. Then graph the equation.



Example 3 *Drawing Appropriate Scales*

Graph the equation $y = 5x + 35$.

Find the intercepts by substituting __ for y and then __ for x.

$y = 5x + 35$

__ $= 5x + 35$

______ $= 5x$

_____ $= x$

The x-intercept is _____ .

$y = 5x + 35$

$y = 5($__$) + 35$

$y =$ ____

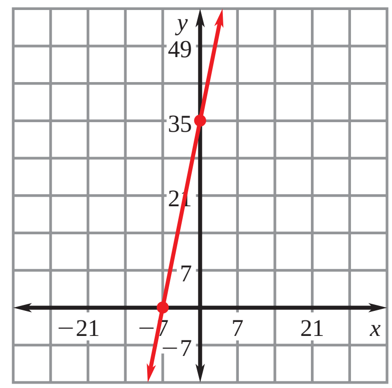
The y-intercept is ____.

Draw a coordinate plane that includes the points (_____, __) and (__ , ____). With these values, it is reasonable to use tick marks at 7-unit intervals.

When making a quick graph, find the intercepts before drawing the coordinate plane. This will help you find an appropriate scale on each axis.

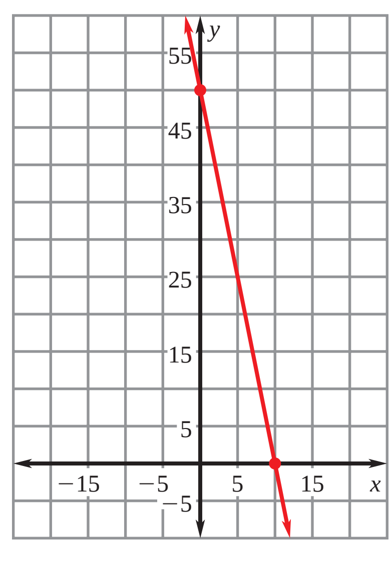
You may want to draw axes with at least two tick marks to the left of -7 and to the right of 0 on the x-axis and two tick marks below 0 and above 35 on the y-axis.

Plot the points (_____, __) and (__ , ____) and draw a line through them.



Checkpoint Complete the following exercise.

2. Graph the equation $y = -5x + 50$.



4.4 The Slope of a Line

Goals
- Find the slope of a line using two of its points.
- Interpret slope as a rate of change in real-life situations.

VOCABULARY

Slope

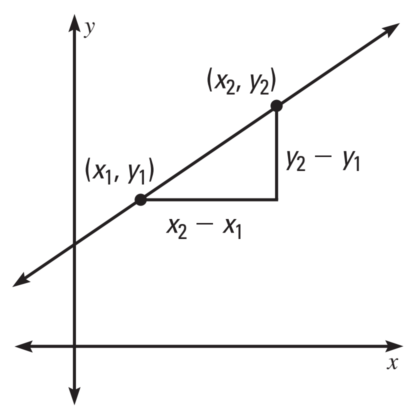
Rate of change

FINDING THE SLOPE OF A LINE

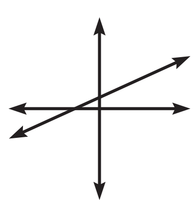
The slope m of the nonvertical line passing through the point (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\square}{\square}$$

$$= \frac{\square}{\square}$$

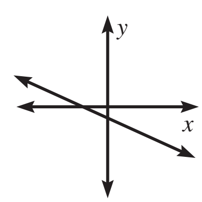


CLASSIFICATION OF LINES BY SLOPE

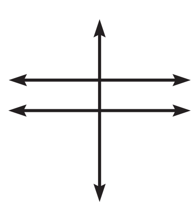


______

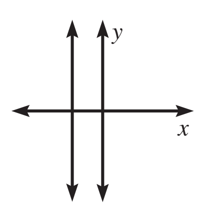
______



A line with a ______ slope ______ from left to right.



A line with ______ slope is ______.



A line with ______ slope is ______.

Example 1 *Finding the Slope of a Line*

Find the slope of the line passing through the points. Then classify the line by its slope.

a. $(-2, -3), (1, 2)$ **b.** $(-2, -3), (4, -3)$ **c.** $(-1, -4), (-1, -2)$

Solution

a. Let $(x_1, y_1) = (\_\_\_\_, \_\_\_\_)$ and $(x_2, y_2) = (\_\_\_, \_\_\_)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **Formula for slope.**

$= \dfrac{\quad}{\_\_\_\_\_\_\_\_}$ **Substitute values.**

$= \dfrac{\quad}{\_\_\_\_\_\_\_\_}$ **Simplify.**

The slope of the line is _________, so the line ______ from left to right.

b. Let $(x_1, y_1) = (\_\_\_\_, \_\_\_\_)$ and $(x_2, y_2) = (\_\_\_, \_\_\_\_)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **Formula for slope.**

$= \dfrac{\quad}{\_\_\_\_\_\_\_\_}$ **Substitute values.**

$= \dfrac{\quad}{\_\_\_\_\_\_\_\_}$ **Simplify.**

The slope of the line is _____, so the line is ___________.

c. Let $(x_1, y_1) = (\_\_\_\_, \_\_\_\_)$ and $(x_2, y_2) = (\_\_\_\_, \_\_\_\_)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **Formula for slope.**

$= \dfrac{\quad}{\_\_\_\_\_\_\_\_}$ **Substitute values.**

$= \dfrac{\quad}{\_\_\_\_\_\_\_\_}$ **Simplify.**

The slope of the line is ___________, so the line is _________.

Checkpoint Find the slope of the line passing through the points. Then classify the line by its slope.

1. $(-5, 2), (4, -1)$	**2.** $(6, 2), (9, 2)$
3. $(-7, 0), (-7, 8)$	**4.** $(2, -4), (8, 6)$

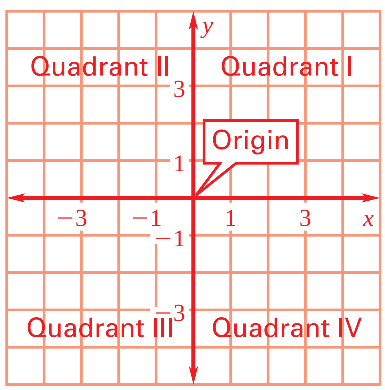
Words to Review

Give an example of the vocabulary word.

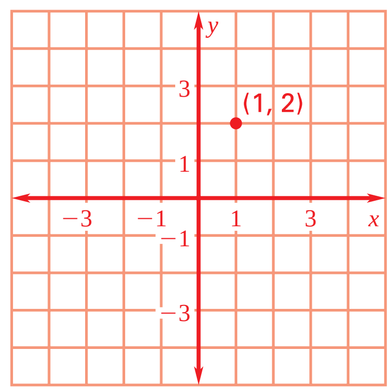
Ordered pair

x-coordinate, *y*-coordinate

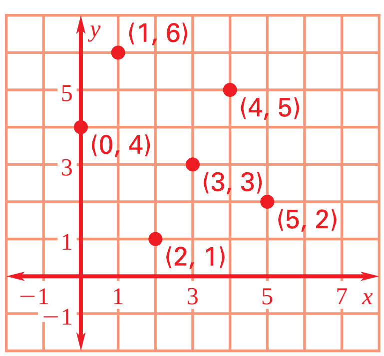
Coordinate plane, origin, quadrants, *x*-axis, *y*-axis



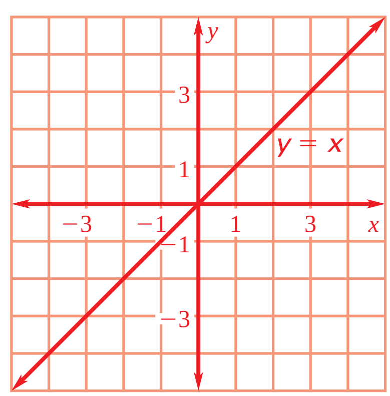
Graph of an ordered pair

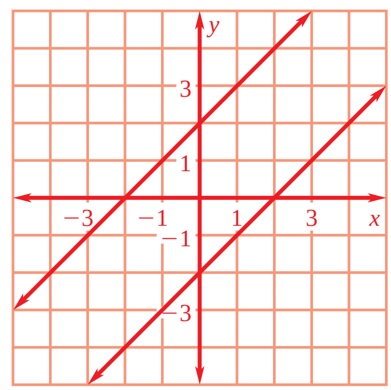
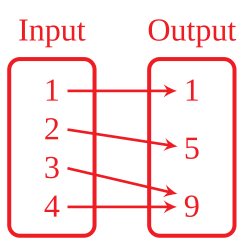
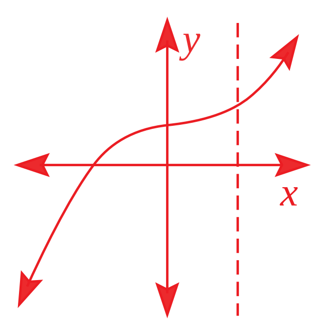


Scatter plot



Graph of a linear equation



Solution of an equation	**x-intercept, y-intercept**
Slope	**Slope-intercept form**
Direct variation, constant of variation	**Parallel lines** 
Relation 	**Graph of a function** 
Function notation	

Review your notes and Chapter 4 by using the Chapter Review on pages 264–266 of your textbook.

4.7 Solving Linear Equations Using Graphs

Goals
- Solve a linear equation graphically.
- Use a graph to solve real-life problems.

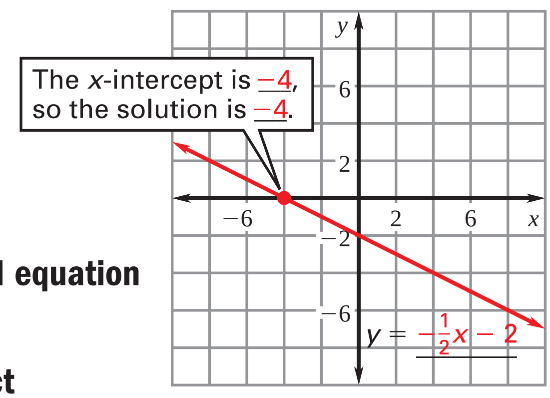
STEPS FOR SOLVING LINEAR EQUATIONS GRAPHICALLY

Step 1 Write the equation in the form ________.
Step 2 Write the related function ________.
Step 3 Graph the equation ________.

The solution of ________ is the ________ of ________.

Example 1 *Solving an Equation Graphically*

Solve $\frac{5}{2}x - 2 = 3x$ graphically.



1. Write the equation in the form $ax + b = 0$.

$\frac{5}{2}x - 2 = 3x$ **Original equation**

________ $= 0$ **Subtract ____ from each side.**

2. Write the related function. $y =$ ________

3. Graph the equation $y =$ ________.

Answer The x-intercept is ____, so the solution is $x =$ ____.

Check Check your answer algebraically.

$\frac{5}{2}x - 2 = 3x$ **Original equation**

$\frac{5}{2}(\_\_\_\_) - 2 \stackrel{?}{=} 3(\_\_\_\_)$ **Substitute ____ for x.**

________ $=$ ____ ______ **statement.**

When you know two ways to solve a problem, it's a good idea to use one method to get a solution and the other method to check the solution.

Example 2 *Using a Graphing Calculator*

Use a graphing calculator to approximate the solution of the linear equation $1.25(2x - 17) = -1.25 + 6.5x$.

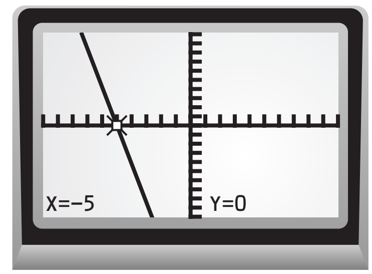
Solution

When you graph an equation using a graphing calculator, you do not need to simplify the equation before entering it.

$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = 0$ **Rewrite equation so one side is 0.**

Then use a graphing calculator (or a computer) to graph the related function

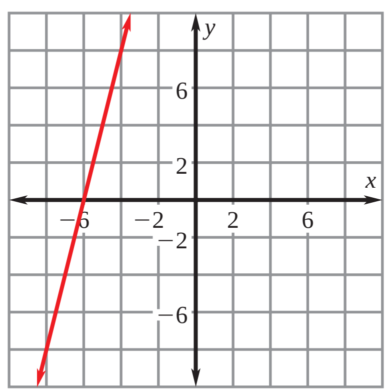
$y = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$.



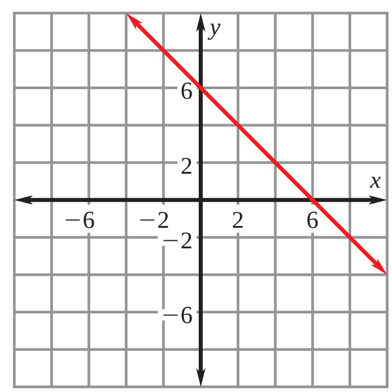
Answer The x-intercept appears to be ____, so the solution is $x =$ ____.

Checkpoint Solve the equation graphically.

1. $3x + 24 = -x$



2. $2x + 6 = 3x$



3. Use a graphing calculator to approximate the solution of the linear equation $2.65(3x - 2) = 6x + 0.55$.

4.8 Functions and Relations

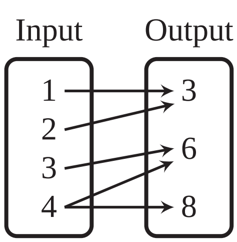
Goals
- Identify when a relation is a function.
- Use function notation to represent real-life situations.

VOCABULARY
Relation
Function notation
Graph of a function

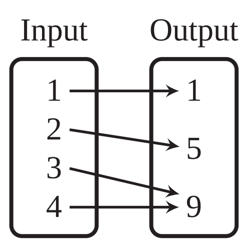
Example 1 *Identifying Functions*

Decide whether the relation is a function. If so, give the domain and the range.

a.



b.



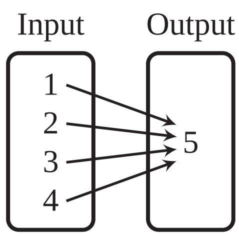
Solution

a. The relation ________ a function because ______________________________ ______________________________.

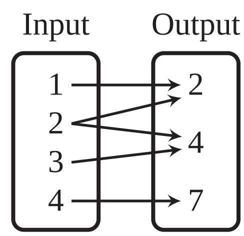
b. The relation ____ a function because ______________________________ ______________________. The domain of the function is the set of ________ values ________________. The range is the set of __________ values ____________.

✔ *Checkpoint* **Decide whether the relation is a function. If so, give the domain and the range.**

1.

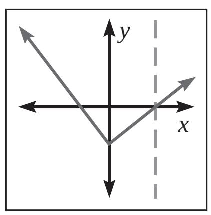


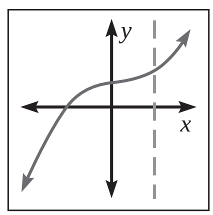
2.

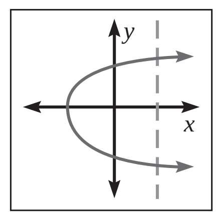


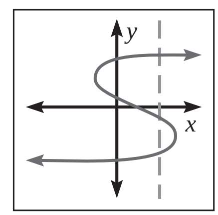
VERTICAL LINE TEST FOR FUNCTIONS

A relation is a function of the horizontal-axis variable if and only if no ________ line passes through ____________________ on the graph.









You don't have to use *f* to name a function. You can use other letters, such as *g* and *h*.

Example 2 ***Evaluating a Function***

Evaluate the function for the given value of the variable.

a. $f(x) = -3x$ when $x = 2$ **b.** $g(x) = 4x + 20$ when $x = -3$

Solution

a. $f(x) = -3x$ **Write original function.**

$f(\_\_\_) = -3(\_\_\_)$ **Substitute ___ for *x*.**

$= \_\_\_\_$ **Simplify.**

b. $g(x) = 4x + 20$ **Write original function.**

$g(\_\_\_\_) = 4(\_\_\_\_) + 20$ **Substitute ____ for *x*.**

$= \_\_\_$ **Simplify.**

Checkpoint Evaluate the function for the given value of the variable.

3. $f(x) = 11x + 3$ when $x = -3$	4. $f(x) = 6 - 1.75x$ when $x = 10$

Example 3 *Graphing a Linear Function*

Graph $f(x) = \frac{3}{4}x - 2$.

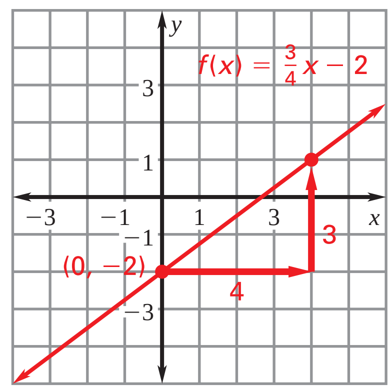
Rewrite the function as ______________.

The *y*-intercept is ____, so plot (0, ____).

The slope is ____.

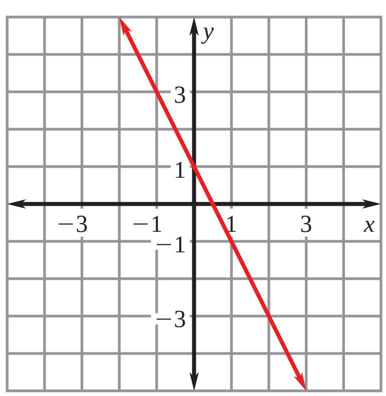
Draw a slope triangle to locate a second point on the line.

Draw a line through the two points.



Checkpoint Graph the function.

5. $f(x) = -2x + 1$



6. $f(x) = 4x - 3$